Sum of Product / Product of Sums

For Sum of Products (SOP)

SOP refers to the fact that terms are OR'ed (summed) after the inputs are AND'ed (product)

- For each output = 1, create the term when AND'ed together will = 1
    - For each input that is a zero use the letter with a bar
    - For each input that is a one use the letter
- OR all the above terms together

A	B	C	Out	Term
0	0	0	1	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

So the full equation is:

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$$

Each term in the expression gives you one of the 1's in the truth table.

Since each input (A, B, C) appears in every term, this equation is said to be in "STANDARD SOP FORM"

For Product of Sums (POS)

POS refers to the fact that terms are AND'ed together (product) after the inputs are OR'ed together (sum).

- For each output = 0, create a term that when OR'ed will be = 0
    - For each input = 1 use the letter with a bar
    - For each input = 0 use the letter
- AND all the above terms together

Opposite of SOP!!

A	B	C	Out	Term
0	0	0	1	
0	0	1	0	$A + B + \bar{C}$
0	1	0	1	
0	1	1	1	
1	0	0	1	
1	0	1	0	$\bar{A} + B + \bar{C}$
1	1	0	0	$\bar{A} + \bar{B} + C$
1	1	1	1	

So the equation becomes:

$$(A + B + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

Proof

First, remember that $F = \bar{\bar{F}}$

So we will use the SOP equation, do Demorgan, then simplify then use Demorgan a 2nd time

(see following pages)

SOP

$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$ orig eqn

$\overline{\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC}$ Demorgan #1

$(\overline{\bar{A}\bar{B}\bar{C}})(\overline{\bar{A}B\bar{C}})(\overline{\bar{A}BC})(\overline{A\bar{B}\bar{C}})(\overline{ABC})$

$(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

$AA + A\bar{B} + AC + AB + B\bar{B} + BC + AC + \bar{B}C + CC$

$A + A\bar{B} + AC + AB + 0 + BC + AC + \bar{B}C + C$

$A(1 + \ldots) + C(\ldots + 1)$

$(A + C)$

$(A+C)(A+\bar{B}+\bar{C})$

$AA + A\bar{B} + A\bar{C} + AC + C\bar{B} + C\bar{C}$

$A + A\bar{B} + A\bar{C} + AC + C\bar{B} + 0$

$A(1 + \ldots) + C\bar{B}$

$(A + C\bar{B})(\bar{A} + B + C)$

$A\bar{A} + AB + AC + \bar{A}C\bar{B} + C\bar{B}B + CC\bar{B}$

$0 + AB + AC + \bar{A}C\bar{B} + 0 + C\bar{B}$

$AB + AC + C\bar{B}(\bar{A} + 1)$

$(AB + AC + C\bar{B})(\bar{A} + \bar{B} + \bar{C})$

$$A\bar{A}B + AB\bar{B} + AB\bar{C} + A\bar{A}C + A\bar{B}C + AC\bar{C} + \bar{A}C\bar{B} + C\bar{B}\bar{B} + C\bar{C}\bar{B}$$

$$0 + 0 + AB\bar{C} + 0 + A\bar{B}C + 0 + \bar{A}C\bar{B} + C\bar{B} \quad 0$$

$$AB\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + \bar{B}C$$

$$AB\bar{C} + \bar{B}C(A + \bar{A} + 1)$$

$$AB\bar{C} + \bar{B}C$$

$$\overline{AB\bar{C} + \bar{B}C}$$ Demorgan #2

$$(\overline{AB\bar{C}})(\overline{\bar{B}C})$$

$$(\bar{A} + \bar{B} + C)(B + \bar{C})$$

but not in std form * next page

POS $$(\bar{A} + \bar{B} + C)(A + B + \bar{C})(\bar{A} + B + \bar{C})$$

Proof

$$(B+\bar{C}) = (A+B+\bar{C})(\bar{A}+B+\bar{C})$$

$$A\bar{A}+AB+A\bar{C}+\bar{A}B+BB+B\bar{C}+\bar{A}\bar{C}+B\bar{C}+\bar{C}\bar{C}$$

$$0 + AB + A\bar{C} + \bar{A}B + B + B\bar{C} + \bar{A}\bar{C} + B\bar{C} + \bar{C}$$

$$B(1 \ldots) + (A\bar{C} + \bar{A}\bar{C} + \bar{C})$$

$$B + \bar{C}(A + \bar{A} + 1)$$

$$B + \bar{C}$$