

Rule 10 proof

$$\begin{array}{l} A + AB = A \\ A(1 + B) = \uparrow \\ \quad \vee \\ A(1) \\ \quad \downarrow \\ A \end{array} \quad \begin{array}{l} \text{Distributive} \\ \#2 \end{array}$$

Rule 11 proof

$$\begin{array}{l} A + \bar{A}B = A + B \\ \overbrace{(A + AB)'} + \bar{A}B \\ A + \overbrace{AB + \bar{A}B} \\ A + B(A + \bar{A}) \\ \quad \vee \\ A + B(1) \\ A + B \end{array} \quad \begin{array}{l} \text{reverse rule \#10} \\ \text{Dist.} \end{array}$$

Rule 12

$$\begin{aligned}(A+B)(A+C) &= A+BC \quad \leftarrow \\ & \text{Dist.} \\ AA+AC+\underline{BA}+BC & \text{commutative} \\ \underline{AA}+AC+AB+BC & \text{rule 7} \\ A+AC+AB+BC & \text{Dist.} \\ A(1+C+B)+BC & \text{rule 2} \\ A(1)+BC & \\ A+BC & \end{aligned}$$

Examples

$$\begin{aligned}W &= ABC + \underline{CAB} + \underline{BAC} \quad \text{comm.} \\ &= ABC + ABC + ABC \quad \text{rule 5} \\ &= ABC \quad \text{(all the same term)} \end{aligned}$$

$$\begin{aligned}X &= ABC + A\overline{B}\overline{C} \quad \text{Dist.} \\ &= AB(C+\overline{C}) \quad \#6 \\ &= AB(\overset{1}{1}) \\ &= AB \end{aligned}$$

$$\begin{aligned}
 Q &= X(\bar{X}yZ + \bar{X}y\bar{Z}) && \text{Dist} \\
 &= \underbrace{X\bar{X}yZ}_0 + \underbrace{X\bar{X}y\bar{Z}}_0 && \begin{array}{l} \text{rule 8} \\ \text{rule 3} \end{array} \\
 &= 0
 \end{aligned}$$

OR

$$\begin{aligned}
 Q &= X(\bar{X}yZ + \bar{X}y\bar{Z}) && \text{Dist.} \\
 &= X(\bar{X}y(Z + \bar{Z})) && \text{rule 6} \\
 &= X(\bar{X}y(1)) && \text{rule 4} \\
 &= X(\bar{X}y) && \text{Assoc.} \\
 &= \underbrace{X\bar{X}}_{(0)}y && \begin{array}{l} \text{rule 8} \\ \text{rule 3} \end{array} \\
 &= 0
 \end{aligned}$$

$$\begin{array}{l}
 a + \bar{a}b \\
 a + b
 \end{array}
 \quad \text{rule 11}$$

$$\begin{aligned}
 &(\bar{a} + \bar{b})(\bar{a} + b) && \text{Dist} \\
 &\bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b && \text{rule 5 + Comm.} \\
 &\underbrace{\bar{a} + \bar{a}b + \bar{a}\bar{b}} + \bar{b}b && \text{Dist.} \\
 &\bar{a}(1 + b + \bar{b}) + \bar{b}b \\
 &\quad \bar{a} + \bar{b}b && \text{rule 8} \\
 &\quad \bar{a} + \underbrace{\bar{b}b}_0 && \text{rule 1} \\
 &\quad A
 \end{aligned}$$

OR

$$\text{rule } (\bar{a} + \bar{b})(\bar{a} + b) = \bar{a} + \bar{b}b \quad \text{rule 12}$$

$$\text{where } A = \bar{a} \\ b = \bar{b} \\ c = b$$

$$\bar{a} + \bar{b}b \quad \text{rule 8}$$

$$\bar{a} + 0 \quad \text{rule 1}$$

$$\bar{a}$$

$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C \quad \text{Dist}$$

$$A\bar{B}C + \bar{A}C(B + \bar{B}) \quad \text{rule 6}$$

$$A\bar{B}C + \bar{A}C \quad \text{Dist}$$

$$C(A\bar{B} + \bar{A}) \quad \text{com}$$

$$C(\bar{A} + A\bar{B}) \quad \text{rule 11}$$

$$C(\bar{A} + \bar{B}) \quad \text{Dist/com}$$

$$\bar{A}C + \bar{B}C$$