



TECH 3232

Digital Technology  
Number Systems

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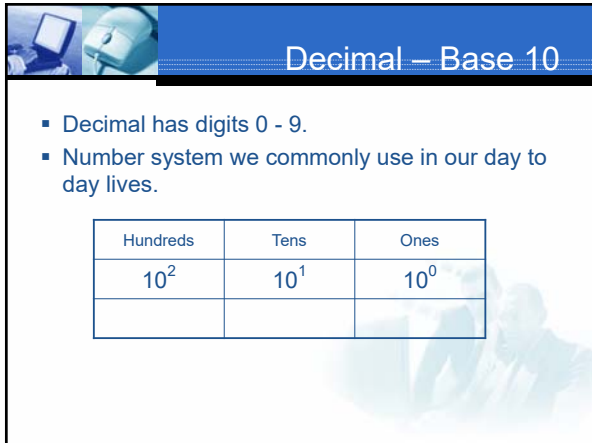
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### Decimal – Base 10

- Decimal has digits 0 - 9.
- Number system we commonly use in our day to day lives.

Hundreds	Tens	Ones
$10^2$	$10^1$	$10^0$

2

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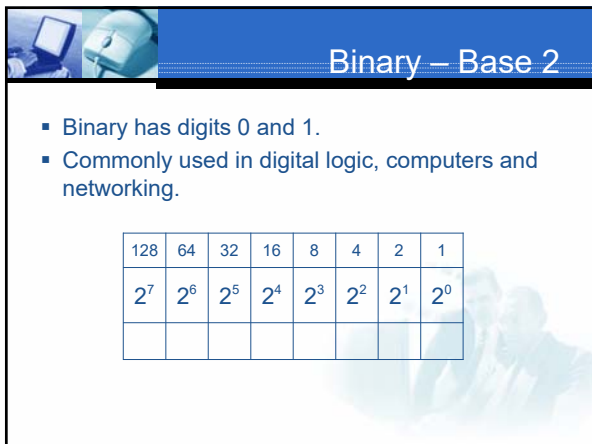
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### Binary – Base 2

- Binary has digits 0 and 1.
- Commonly used in digital logic, computers and networking.

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$

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**Worked Problem**

Convert 111 Dec to Binary

128	64	32	16	8	4	2	1
2 <sup>7</sup>	2 <sup>6</sup>	2 <sup>5</sup>	2 <sup>4</sup>	2 <sup>3</sup>	2 <sup>2</sup>	2 <sup>1</sup>	2 <sup>0</sup>
0	1	1	0	1	1	1	1

$$\begin{array}{r}
 111 \text{ dec} \\
 - 64 \\
 \hline
 47 \\
 - 32 \\
 \hline
 15 \\
 - 8 \\
 \hline
 7 \\
 - 4 \\
 \hline
 3 \\
 - 2 \\
 \hline
 1 = 0
 \end{array}$$


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**Worked Problem**

Convert 210 Dec to Binary

128	64	32	16	8	4	2	1
2 <sup>7</sup>	2 <sup>6</sup>	2 <sup>5</sup>	2 <sup>4</sup>	2 <sup>3</sup>	2 <sup>2</sup>	2 <sup>1</sup>	2 <sup>0</sup>
1	1	0	1	0	1	0	0

$$\begin{array}{r}
 210 \\
 - 128 \\
 \hline
 82 \\
 - 64 \\
 \hline
 18 \\
 - 16 \\
 \hline
 2
 \end{array}$$


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**Convert 100 to binary**  
(division method)

Convert 100 to binary using division method.

Division	Quotient	Remainder

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### Example Binary to Decimal Conversion

Convert  $10101110_2$  to decimal

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$

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### Example Binary to Decimal Conversion

Convert  $11010010_2$  to decimal

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$

8

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### Octal – Base 8

- Octal has digits 0 through 7.
- Used to be used in computers (but rarely used today).
- Why base 8? Because 3 bits can be converted to decimal digits 0 -> 7.

4	2	1
$2^2$	$2^1$	$2^0$

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### Example Octal to Decimal Conversion

Convert 127 Octal to Decimal.

4	2	1	4	2	1	4	2	1
256	128	64	32	16	8	4	2	1

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### Worked Problem

Convert 011010111 binary (215 dec) to Octal

256	128	64	32	16	8	4	2	1
2 <sup>8</sup>	2 <sup>7</sup>	2 <sup>6</sup>	2 <sup>5</sup>	2 <sup>4</sup>	2 <sup>3</sup>	2 <sup>2</sup>	2 <sup>1</sup>	2 <sup>0</sup>
0	1	1	0	1	0	1	1	1
4	2	1	4	2	1	4	2	1
	3		2			7		

*215 dec*

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### Worked Problem

Convert 001010110 binary to dec (86) and Octal (126)

256	128	64	32	16	8	4	2	1
2 <sup>8</sup>	2 <sup>7</sup>	2 <sup>6</sup>	2 <sup>5</sup>	2 <sup>4</sup>	2 <sup>3</sup>	2 <sup>2</sup>	2 <sup>1</sup>	2 <sup>0</sup>
0	0	1	0	1	0	1	1	0
4	2	1	4	2	1	4	2	1
		1		2		6		

*64  
16  
4  
2  
86*

12

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**Worked Problem**

Binary 010100110 back to dec 166 and to octal 246

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**Example Octal to Decimal Conversion**

Convert 476 Octal to Decimal.

4	2	1	4	2	1	4	2	1
256	128	64	32	16	8	4	2	1

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**Hex – Base 16**

If Binary (base 2) uses digits 0 and 1 and Octal (base 8) uses digits 0 though 7.

What would Base 16 use?

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## Hex – Base 16

If Binary (base 2) uses digits 0 and 1 and Octal (base 8) uses digits 0 though 7.

What would Base 16 use?

But we represent 10 though 15 as “A” though “F”

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## Hex – Base 16

Base 10	Base2	Base 16	Base 10	Base2	Base 16
0	0000	0	8	1000	8
1	0001	1	9	1001	9
2	0010	2	10	1010	A
3	0011	3	11	1011	B
4	0100	4	12	1100	C
5	0101	5	13	1101	D
6	0110	6	14	1110	E
7	0111	7	15	1111	F

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## Hex – Base 16

- Most commonly used in computers and networking (error messages in windows and mac addressing)
- Why base 16? Because 4 bits can be converted to decimal digits 0 -> 15.

8	4	2	1
$2^3$	$2^2$	$2^1$	$2^0$

18

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### Example Decimal to Hex Conversion

Convert 100 to Hex via Binary.

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
8	4	2	1	8	4	2	1

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### Worked example

Binary 11010101 to Hex

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	1	0	1	0	1	0	1

13    D    5

A=10 B=11 C=12 D=13 E=14 F=15

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### Worked Example

Binary 01111110 to hex

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
0	1	1	1	1	1	0	

7    E    14

A=10 B=11 C=12 D=13 E=14 F=15

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## Binary Coded Decimal

- Binary Coded Decimal (BCD for short) is a way of storing decimal digits in a binary format.
- Each nibble (4 bits) is used to store a digit 0-9 of the decimal value.



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## Example Decimal to BCD Conversion

Convert 97 Decimal to BCD.

128	64	32	16	8	4	2	1
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
8	4	2	1	8	4	2	1

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## Worked Example

Dec 147 to BCD

dec value 147

$$\begin{array}{r|l} 8 & 4 & 2 & 1 & 8 & 4 & 2 & 1 \\ \hline \rightarrow & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{array} \leftarrow \text{BCD}$$

DEC 147 and Binary

$$\begin{array}{r} 128 \\ 147 \\ -128 \\ \hline 19 \\ 16 \\ \hline 3 \\ -3 \\ \hline 0 \end{array}$$

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## Worked Example

BCD 0010 0010 0001 to Dec, Binary, and Hex

$$\begin{array}{r} \begin{array}{cccc} 0010 & 0010 & 0001 & \text{BCD} \\ 2 & 2 & 1 & \text{dec} \end{array} \\ \\ \begin{array}{r} \begin{array}{cccc|cccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ \hline 6 & 4 & 2 & 1 & 8 & 4 & 2 & 1 \end{array} \\ \begin{array}{cc} D & D \end{array} \end{array} \\ \\ \begin{array}{r} 221 \\ -128 \\ \hline 93 \\ -64 \\ \hline 29 \\ -16 \\ \hline 13 \\ \hline 55 \end{array} \end{array}$$

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